Age-Period-Cohort Detrended APC-D model

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Abstract: *In the Age-Period-Cohort APC tradition (Mannheim, 1928; Ryder, 1965; Mason et al. 1973; Hobcraft et al. 1982; Yang et al. 2004; O'Brien 2011) the detection of "intrinsic" cohort effects is a general aim. Here, a better APC method is proposed in order to make the difference between "linear trends" and fluctuations. To do so, the APC-D (detrended) model delivers a DCE (detrended cohort effect, having a zero-sum zero-slope shape) set of parameters.*

Expression of the model

An OLS specification[1] of the APCD model is presented here: we consider a dependant variable y, for instance the logged annual income, in a microdata set of independent cross-sectional surveys. Here, y_i^{apc} pertains to individual i of age a in period p, and thus belonging to the birth cohort c=p-a. The intervals $[a_{min},a_{max}]$ and $[p_{min},p_{max}]$ denotes respectively the age-span and the periods of observation defined by a series of cross-sectional independent sample surveys carried with a regular pace p (yearly, or each 5 years, for example); the age groups a are based on the same pace of time than periods; we will suppose no hole in the a p rectangle. We exclude the first and the last cohorts of the estimations of the models, in order to improve the confidence intervals of the parameters. Then, c is in the interval $[p_{min}-a_{max}+1,\ p_{max}-a_{min}-1]$. Control variables (continuous or dichotomic ones) X_j, such as gender, race, education, state, etc. are to be included as covariates.

The APCD separates linear trends on the one hand and fluctuations on the other. The linear trends of age, period and cohort can not be disentangled properly within exceptions because of the linear relation a=p-c. Thus, the linear trend part of the model must be considered as blank linear parameters. Besides this problem, the model performs a unique decomposition of age, period and cohort fluctuations around the trend with the inclusion of appropriate constraints where the pertaining sets of age, period and cohort parameters have a zero-sum and a zero-slope that solve the traditional identification problem[2] of the APC (Wilmoth, 2001, Chauvel, 2001). In general, we focus at first on the DCE (detrended cohort effect, having a zero-sum zero-slope shape) which express the "cohort effect" defined as the cohort fluctuation component of the variable. The test of the difference to zero of the DCE is the central aspect of the cohort problem.

$$(0)\begin{cases} y^{apc} = \alpha_a + \pi_p + \gamma_c + \alpha_0 rescale(a) + \gamma_0 rescale(c) + \beta_0 + \sum_j \beta_j X_j + \varepsilon_i \\ \begin{cases} p = c + a \\ \sum_a \alpha_a = \sum_p \pi_p = \sum_c \gamma_c = 0 \\ Slope_a(\alpha_a) = Slope_p(\pi_p) = Slope_c(\gamma_c) = 0 \\ \min(c) < c < \max(c) \end{cases} \end{cases} \qquad \text{(APCD)}$$

- The set of constraints –on the zero-sum, zero-slopes and on the domain of estimation of the cohort effects that excludes the first and the last cohort– produce a unique estimate of DCE and solve the old APC identification problem.
- Rescale(a) is the linear function that rescales the index a (age) from -1 to 1.
- $Slope_a(\alpha_a)$ is the linear slope of the α_a estimates. $Slope_a(\alpha_a)=0$ if and only if

  $\Sigma_a [(2a - a_{min} - a_{max})\ \alpha_a] = 0$
- α_a, π_p and γ_c are respectively the detrended age, period and cohort effects. The π_p effects fit the categorical period changes, and are able to absorb the period-specific

[1] The GLM procedure of STATA offers a wide range of specification from OLS to logit or Poisson models of different kinds. Unfortunately ordinal or polytomic logit models, and also quantile regressions, can not be easily handled.

[2] Several solutions have been proposed, such as the Yang Yang and colleagues APC-IE model which generally performs accurate results. The only problem of APC-IE is the production of unstable cohort estimates since their slope is unstable when one changes the metrics of the dependant variable. For instance, the estimated APC-IE cohort effects are different whether one considers nominal wages or real ones when the APCD provides a unique DCE since the difference between real and nominal wages is absorbed by the period coefficients.

changes in measurements of the dependant variable, effects of inflation, etc. The α_a represent the non-linear age changes. For our purpose, γ_c (also named "detrended cohort effects" DCE) are the most important estimates of this model since significantly non-zero γ_c coefficients will detect cohort effects.

- β_0 is the general intercept, and $\beta_j X_j$ pertain to the controls by selected covariates such as gender, race, education, etc. that can be introduced in the model.
- α_0 is the interperiod, intercohort linear slope of age. γ_0 is the interperiod interage linear slope of cohort. Since we have a linear relation p=a+c, these two coefficients are not to be naively analysed in terms of age and cohort effects but as blank linear intercepts.

In this APCD model, if no cohort effect $\hat{\gamma}_c$ is significantly different to 0, the simple age and period AP model is sufficient representation of data. Raftery's (1986) BIC could help to decide between AP and APCD. Conversely, when at least one of the DCE(0) estimates is significantly different to 0, the pertaining cohort effect can be defined as an average specific behavior for the cohort, averaged on the available age-span.

References

Chauvel L., 2001, "Education and Class Membership Fluctuations by Cohorts in France and United-States (1960-2000)", presented at the Mannheim workshop of the International sociological association Research committee 28 (Stratification and social mobility), mimeo, http://www.louischauvel.org/MANNHEIM.pdf

Hobcraft J., J. Menken, and S. Preston. 1982, "Age, Period, and Cohort Effects in Demography: A Review." *Population Index* 48:4–43.

Mannheim K.,1928, "Das Problem der Generationen", *Kölner Vierteljahres Hefte für Soziologie* 7: 157-85, 309-30.

O'Brien R., 2011, "Constrained estimators and age-period-cohort models." *Sociological Methods & Research,* 40: 419-452.

Raftery A.E., 1986, "Choosing models for cross-classifications", *American Sociological Review*, 51, 145-146.

Ryder N.B., 1965, "The Cohort as a Concept in the Study of Social Change", *American Sociological Review*, 30: 843-861.

Wilmoth J.R., 2001, "Les modèles âge-période-cohorte en démographie." In: G. Caselli, J. Vallin and G. Wunsch (eds.), *Démographie: Analyse et Synthèse. I. La Dynamique des Populations*, Paris: INED, pp. 379-397.

Yang Y., S. Schulhofer-Wohl, W. Fu, and K. Land. 2008. "The Intrinsic Estimator for Age-Period-Cohort Analysis: What It is and How to Use it?" *American Journal of Sociology*, 113: 1697-1736.